

# Program Description I

Program Title GRAECO-LATIN SQUARE

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**Program Description, Equations, Variables** A Graeco-Latin square permits a three-way control in the variation of experimental units, i.e. row effects, column effects and "layer" effects. A Graeco-Latin square defines a restricted randomization procedure whereby  $p$  treatments are assigned to  $p^2$  experimental units so as to obtain balance along three dimensions. From the point of view of construction such a square is a composite of two orthogonal Latin squares. Independent randomization of rows and columns is required before the resulting composite square is used in practice. In order to maintain the identity of the squares which are combined to form the composite, the cells of one square are designated by Latin letters, and the cells of the second square by Greek letters. Thus for two  $3 \times 3$  squares

1	2	Composite																											
<table border="1"> <tr><td>a</td><td>b</td><td>c</td></tr> <tr><td>b</td><td>c</td><td>a</td></tr> <tr><td>c</td><td>a</td><td>b</td></tr> </table>	a	b	c	b	c	a	c	a	b	<table border="1"> <tr><td><math>\alpha</math></td><td><math>\beta</math></td><td><math>\gamma</math></td></tr> <tr><td><math>\gamma</math></td><td><math>\alpha</math></td><td><math>\beta</math></td></tr> <tr><td><math>\beta</math></td><td><math>\gamma</math></td><td><math>\alpha</math></td></tr> </table>	$\alpha$	$\beta$	$\gamma$	$\gamma$	$\alpha$	$\beta$	$\beta$	$\gamma$	$\alpha$	<table border="1"> <tr><td><math>a\alpha</math></td><td><math>b\beta</math></td><td><math>c\gamma</math></td></tr> <tr><td><math>b\gamma</math></td><td><math>c\alpha</math></td><td><math>a\beta</math></td></tr> <tr><td><math>c\beta</math></td><td><math>a\gamma</math></td><td><math>b\alpha</math></td></tr> </table>	$a\alpha$	$b\beta$	$c\gamma$	$b\gamma$	$c\alpha$	$a\beta$	$c\beta$	$a\gamma$	$b\alpha$
a	b	c																											
b	c	a																											
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$c\beta$	$a\gamma$	$b\alpha$																											

The sample problem given illustrates the use for a  $4 \times 4$  Graeco Latin square.

The equations used are:

$$1. \text{ Row Sums} = RS_i = x_i, i=1,2,\dots,n \quad (a)$$

$$2. \text{ Col Sums} = CS_j = x_j, j=1,2,\dots,n \quad (b)$$

$$3. \text{ TR}_1 \text{ Sums} = TR_1 = x_{tr1}, 1,2,\dots,n \quad (c)$$

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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$$4. \quad TR_2 \text{ Sums} = \quad TR_2 = \quad x_{Tr2} = 1, 2, \dots, n \quad (d)$$

$$5. \quad \text{Total Sums} = \quad (a) + (b) = (e)$$

The sum of squares for each of the above equations  
 $= (a)^2, (b)^2, (c)^2, (d)^2, \text{ and } (e)^2$  respectively.

## Variations.

$$6. \quad \text{Row Variations} = \frac{(a)^2}{n^2} \quad (f)$$

$$7. \quad \text{Col Variations} = \frac{(b)^2}{n^2} \quad (g)$$

$$8. \quad \text{Treatment 1} = \frac{(c)^2}{n^2} \quad (h)$$

$$9. \quad \text{Treatment 2} = \frac{(d)^2}{n^2} \quad (i)$$

$$10. \quad \text{Total} = \frac{(e)^2}{n^2} \quad (j)$$

$$11. \quad \text{Error variation} = (j) - (f) - (g) - (h) - (i)$$

where  $n$  = no. of rows

$$11. \quad \text{Degrees of freedom for error} = (n^2 - 1) - 4(n-1)(n-3)$$

$$12. \quad \text{Mean Square} = \frac{\text{Variation}}{df}$$

$$13. \quad F \text{ ratio} = \frac{\text{Mean Square}}{df - 1}$$

Correction of erroneous data input. Note that there is no automatic correction label. If an error is input and it is not desired to start afresh summing the data the following routine will delete the error - Re-enter the erroneous data and do (Sto - 1) (x<sup>2</sup>) (Sto-6) (Rcl 2) (1)(-)(Sto 2). The program can then continue in the normal way.

Note also that this correction process is easily carried out with all data entries except the final one (to complete col. row etc. summing) This is because the program automatically halts. In the latter case advantage must be taken of the pauses to press (R/S) and then the correction routine. Otherwise the program automatically proceeds to the next step at this stage and you will therefore miss the opportunity of correcting the erroneous input. That is the only reason for the inclusion of the "pauses" in the program.

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Problem: In the following table A,B,C,D represent four different types of gasoline, the Greek letters denote different roads used, and the numbers represent mileages obtained in an experiment using four different drivers in four different cars. Use a Graeco-Latin square design to ascertain by analysis of variance if differences exist between the various types of gasoline.

		Drivers			
		1	2	3	4
C a r s	1	B $\gamma$ 19	A $\beta$ 16	D $\zeta$ 16	C $\alpha$ 14
	2	A $\zeta$ 15	B $\alpha$ 18	C $\gamma$ 11	D $\beta$ 15
	3	D $\alpha$ 14	C $\zeta$ 11	B $\beta$ 21	A $\gamma$ 16
	4	C $\beta$ 16	D $\gamma$ 16	A $\alpha$ 15	B $\zeta$ 23

Solution: Load Card (both sides)

Enter No. of rows 4 (A)

Sum row mileages - 19 (E) 16 (E) 16 (E) 14 (E) output = 65  
 15 (E) 18 (E) 11 (E) 15 (E) output = 59  
 14 (E) 11 (E) 21 (E) 16 (E) output = 62  
 16 (E) 16 (E) 15 (E) 23 (E) output = 4

Sum col. mileages= 19 (E) 15 (E) 14 (E) 16 (E) output = 64  
 16 (E) 18 (E) 11 (E) 16 (E) output = 61  
 16 (E) 11 (E) 21 (E) 15 (E) output = 63  
 14 (E) 15 (E) 16 (E) 23 (E) output = 4

Sum Latin letters A's 15 (E) 16 (E) 15 (E) 16 (E) output = 62  
 B's 19 (E) 18 (E) 21 (E) 23 (E) output = 81  
 C's 16 (E) 11 (E) 11 (E) 14 (E) output = 52  
 D's 14 (E) 16 (E) 16 (E) 15 (E) output = 4

Sum Greek letters  $\alpha$ 's 14 (E) 18 (E) 15 (E) 14 (E) output = 61  
 $\beta$ 's 16 (E) 16 (E) 21 (E) 15 (E) output = 68  
 $\gamma$ 's 19 (E) 16 (E) 11 (E) 16 (E) output = 62  
 $\zeta$ 's 15 (E) 11 (E) 16 (E) 23 (E) output = 11st

see next page

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## Variations

16.50 \*\*\*  
 6.50 \*\*\*  
 111.50 \*\*\*  
 7.50 \*\*\*  
 6.00 \*\*\*  
 148.00 \*\*\*

## Degrees of Freedom

3.00 \*\*\*  
 3.00 \*\*\*  
 3.00 \*\*\*  
 3.00 \*\*\*  
 3.00 \*\*\*

## Mean Squares

5.50 \*\*\*  
 2.17 \*\*\*  
 37.17 \*\*\*  
 2.50 \*\*\*  
 2.00 \*\*\*

## F Ratios

2.75 \*\*\*  
 1.00 \*\*\*  
 18.58 \*\*\*  
 1.25 \*\*\*

These may be conveniently set up in the following tableau form:

	Variation	D/F	Mean Square	F ratio
Cars	16.50	3.00	5.50	2.75
Drivers	6.50	3.00	2.17	1.00
Gasolines	111.50	3.00	37.17	18.58
Roads	7.50	3.00	2.50	1.25
Error	6.00	3.00	2.00	
Total	148.00			

Sketch(es)

Sample Problem(s) (continued)

We have therefore Critical  $F_{.95,3,3} = 9.28$

and Critical  $F_{.99,3,3} = 29.50$

Thus we can reject the hypothesis that the gasolines are the same at the 0.05 level but not at the 0.01 level.

Solution(s)

Reference(s) Probability and Statistics (Murray R. Spiegel) Schaum's Outline Series p. 328 ff.

For a detailed treatment of this subject see "Statistical Principles in Experimental Design" by B.J. Winer (McGraw Hill - 1971) Chapter 9.

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# User Instructions

GRAECO LATIN SQUARE

1

2

dp

n

Input

[illegible]

# Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	001	*LBLE	21 11	057			
	002	CFB	16 22 02	058	ST+5	35-55 03	
	003	ENT	-21	059	ROL	36 01	
	004	CLRS	16-53	060	ST0-	35 04	
	005	P25	16-51	061	ST+5	35-55 05	
	006	CLRS	16-53	062	ROL	36 02	
	007	ST00	35 14	063	ROL	36 15	
	008	X2	53	064	+	-55	
	009	P25	16-51	065	ST0E	35 15	
010	010	ST00	35 09	066	ROL2	36 02	
	011	P25	16-51	067	2	00	
	012	1	01	068	ST01	35 01	
	013	0	00	069	ST02	35 02	
	014	ST01	35 46	070	CLN	-51	
	015	ROL0	36 14	071	ROL4	36 04	
	016	RTN	24	072	ROL	36 15	
	017	*LBLE	21 00	073	P25	16-51	
	018	0	00	074	ROL5	36 09	
	019	ST01	35 01	075	P25	16-51	
020	020	ST02	35 02	076	X=Y0	16-33	
	021	ST03	35 03	077	ST08	22 08	
	022	ST05	35 05	078	R+	-31	
	023	ST06	35 06	079	R+	-31	
	024	ST0E	35 15	080	R+5	51	
	025	P25	16-51	081	*LBLE	21 08	
	026	2	02	082	F00	16 23 00	
	027	ST+7	35-55 07	083	ST00	22 13	
	028	ROL7	36 07	084	F10	16 23 01	
	029	ROL5	36 09	085	ST00	22 14	
030	030	P25	16-51	086	ST02	22 02	
	031	X=Y0	16-33	087	*LBLE2	21 02	
	032	ST06	22 16 11	088	SF0	16 21 00	
	033	ROL0	36 14	089	SF1	16 21 01	
	034	RTN	24	090	ST03	22 03	
	035	*LBLE	21 15	091	*LBLE	21 04	
	036	ST+1	35-55 01	092	ROL6	36 06	
	037	X2	53	093	ROL5	36 05	
	038	ST+5	35-55 06	094	X2	53	
	039	1	01	095	R+	16-31	
040	040	ST+2	35-55 02	096	ST02	35 02	
	041	ROL	36 02	097	+	-24	
	042	ROL	36 14	098	ST01	35 01	
	043	X=Y0	16-33	099	-	-45	
	044	ST05	22 12	100	ST00	35 00	
	045	R+	16-31	101	RTN	24	
	046	CLN	54	102	*LBLE	21 13	
	047	R+5	51	103	SF1	16 21 01	
	048	*LBLE	21 12	104	CFB	16 22 00	
	049	R+	16-31	105	GS25	23 06	
050	050	CLN	54	106	ST07	35 07	
	051	P25	16 51	107	GS27	23 07	
	052	P25	16 51	108	*LBLE	21 04	
	053	P25	16 51	109	ST01	35 08	
	054	ROL	36 01	110	ST03	22 00	
	055	X2	53	111	*LBLE	21 14	
	056	ROL	36 02	112	ST07	16 17 13	

REGISTERS										
0 USED	1 USED	2 USED	3 USED	4 USED	5 USED	6 USED	7 USED	8 USED	9 USED	
S0 USED	S1 USED	S2 USED	S3 USED	S4 USED	S5 USED	S6 USED	S7 USED	S8	S9 $n^2$	
A		B		C		D $n$		E USED		I COUNTER

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## Program Listing II

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
113	CF1	16 22 01		169	RCLE	36 06	
114	GSEB	23 06		170	GSEB	23 16 14	
115	GSEB	23 07		171	RCLE	36 06	
116	ST09	35 09		172	GSEB	23 16 14	
117	GT00	22 00		173	RCLE	36 06	
118	*L5L7	21 07	INDIVIDUAL SS.	174	GSEB	23 16 14	
119	RCLE	36 03		175	RCLE	35 06	
120	RCLE	36 01		176	GSEB	23 16 14	
121	-	-45		177	RCLE	36 00	
122	ST-7	35-45 07		178	RCLE	36 06	
123	ST04	35 04		179	=	-24	
124	*L5L5	21 05	TEST FOR EMPTY	180	ST00	35 00	
125	RCLE	36 45	STORAGE REGISTER	181	GSEB	23 16 14	
126	X=02	16-43		182	RCLE	36 01	
127	GT03	22 03		183	RCLE	36 06	
128	ISZ1	16 26 46	LOOP TO TEST	184	=	-24	
129	GT05	22 05		185	ST01	35 01	
130	*L5L3	21 03	STORE SS. IN	186	GSEB	23 16 14	
131	RCLE	36 04	NEXT AVAILABLE	187	RCLE	36 02	
132	ST01	35 45	REGISTER	188	RCLE	36 06	
133	GT04	22 04		189	=	-24	
134	*L5L4	21 16 14	PRINT ROUTINES	190	ST02	35 02	
135	PRTY	-14		191	GSEB	23 16 14	
136	SFC	16-11		192	RCLE	36 03	
137	RTH	24		193	RCLE	36 06	
138	*L5L6	21 16 11	COMPUTE ANOVA	194	=	-24	
139	RCLE	36 00	TABLE	195	ST03	35 03	
140	P22	16-51		196	GSEB	23 16 14	
141	ST05	35 05		197	RCLE	36 04	
142	RCLE	36 00		198	RCLE	36 06	
143	-	-45		199	=	-24	
144	RCLE	36 01		200	GSEB	23 16 14	
145	-	-45		201	RCLE	36 06	
146	RCLE	36 02		202	=	01	
147	-	-45		203	-	-45	
148	RCLE	36 03		204	ST06	35 06	
149	-	-45		205	RCLE	36 00	
150	ST04	35 04		206	X2Y	-41	
151	RCLE	36 14		207	=	-24	
152	=	01		208	GSEB	23 16 14	
153	-	-45		209	RCLE	36 01	
154	ST05	35 05		210	RCLE	36 06	
155	RCLE	36 00		211	=	-24	
156	GSEB	23 16 14		212	GSEB	23 16 14	
157	RCLE	36 01		213	RCLE	36 02	
158	GSEB	23 16 14		214	RCLE	36 06	
159	RCLE	36 02		215	=	-24	
160	GSEB	23 16 14		216	GSEB	23 16 14	
161	RCLE	36 03		217	RCLE	36 03	
162	GSEB	23 16 14		218	RCLE	36 06	
163	RCLE	36 04		219	=	-24	
164	GSEB	23 16 14		220	GT04	22 16 14	TO LAST PRINTOUT
165	RCLE	36 05		221	R/S	51	
166	GSEB	23 16 14					
167	RCLE	36 06					
168	GSEB	23 16 14					

LABELS				FLAGS		SET STATUS		
A INPUT N	B $\Sigma$ Rate	C CONTROL	D	E INPUT DATA	F CONTROL	FLAGS	TRIG	DISP
a TABLE LIST	b	c	d PRINT	e	1 CONTROL	ON OFF		
0 CLEAR REG	1	2 FLAG RESET	3 SEC. REG STORAGE	4 CONTROL	2 CONTROL	0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
5 STO. SUMS LOOP	6 $\Sigma$ SS TOTAL	7 $\Sigma$ SS IND	8 CONTROL	9	3	1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
						2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
						3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n 2